

Neutrino processes with power law dispersion relations

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Abstract

We compute various processes involving neutrinos in the initial and/or final state and we assume that neutrinos have energy momentum relation with a general power law $E^2 = p^2 + \xi_n p^n$ correction due to Lorentz invariance violation. We find that for $n > 2$ the bounds on ξ_n from direct time of flight measurement are much more stringent than from constraining the neutrino Cerenkov decay process.

1 Introduction

The OPERA observation [1] of superluminal neutrinos has been ruled out by the measurement by ICARUS [2] which puts an upper bound on the superluminality of neutrinos at $\delta < 2.3 \times 10^{-7}$. An phenomenological consequence of neutrinos being superluminal was the observation by Cohen and Glashow (GC)[3] and others [4, 5, 6, 7] that superluminal neutrinos are kinematically allowed to emit pairs of e^+e^- and would thereby lose most of their energy during the CERN-Gran Sasso flight of 730 km [3]. ICARUS experiment [8] searched for the Cerenkov emission of electrons in the same CERN-CNGS to Gran Sasso neutrino beams, where according to the GC calculation, 63% of the neutrinos are expected to decay and no anomalous e^+e^- events were detected. Using this a much stronger bound $\delta < 2.5 \times 10^{-8}$ can be put on the superluminal neutrinos. Another problem with superluminal neutrinos which has been discussed [9, 4, 10, 11] is that the change in neutrino energy-momentum relation restricts the phase-space for the $\pi \rightarrow \nu\mu$ process and the pion lifetime would be larger for the OPERA neutrinos produced by the pion decay from the CERN-CNGS beam.

In a measurement by MINOS [12] it was found that muon neutrinos of average energy 3 GeV traversing a distance 730 km exceed c by an amount, $\delta(E = 3\text{GeV}) = (5.1 \pm 2.9) \times 10^{-5}$. This however is in contrast to the neutrino observations from supernova SN 1987a [13, 14] where over a flight path of 51 kpc, the neutrinos with energy in the band (7.5 – 39) MeV all arrived within a time span of 12.4 sec and the optical signal arrived after 4 hours of the neutrino signal (consistent with prediction of supernova models) from which it is inferred that $\delta(E = 15\text{MeV}) \leq 10^{-9}$.

In the present paper we calculate the rates for e^+e^- radiation from neutrinos and pion decay assuming that the neutrinos obey the energy momentum relation of the form $E^2 = m^2 + p^2 + \xi_n p^n$. This is motivated by Horava-Lifshitz type field theories [15, 16] where the higher derivative terms break Lorentz invariance at high scale but help in removing ultraviolet divergence. We find that for $n > 2$ the bounds on ξ_n from direct time of flight measurement [2] are much more stringent than from constraining the Cerenkov decay process during the Cern to Gran-Sasso flight of neutrinos [8]. This is unlike the case for $n = 2$ studied by Glashow-Cohen [3] and [9, 4, 10, 11] where the constraint on Lorentz violation from the kinematically forbidden processes is more stringent than from the time of flight measurement.

Models which explain the Opera result of superluminal neutrinos and which have a bearing on the question of Cerenkov emission from neutrinos or the pion decay kinematics fall broadly in the following categories:

1. Deformed Lorentz symmetry models [17, 18, 19, 20, 21, 22, 23] where the dispersion relations change from the usual $p^2 = m^2$ form to a different form which is still covariant under the modified Lorentz transformations. In this picture the processes which are forbidden in one reference frame (like the rest frame of the massive neutrinos) will be forbidden also in the lab frame.
2. Lorentz invariance violation as in Lifshitz type field theories [15, 16, 24, 25, 26], from a gauge singlet SUSY sector [27] or a hidden sector[28], environmental couplings [29, 30, 31, 32, 33, 34, 35], dynamical symmetry breaking [37, 38], Fermi-point splitting [39], space-time fluctuations [40] and string theory [41].

In this paper we consider the following general dispersion relation motivated by Horava-Lifshitz theories

$$E^2 = m^2 + p^2 + \xi_n p^n \quad (1)$$

where $n = 2, 3, 4..$ etc. The difference between the superluminal neutrino velocity and the speed of light (taken to be 1) is then given by

$$\delta = \frac{\partial E}{\partial p} - 1 \simeq \frac{n-1}{2} \xi_n p^{n-2}, \quad n = 2, 3, 4 \dots \quad (2)$$

The ICARUS time of flight experiment [2] has observed neutrinos and the time difference between the neutrino time of flight (tof) and the calculated photon tof is $\delta t = 0.3 \pm 4.9(stat) \pm 9.3(stat)$ for neutrino energy $E_\nu = 12.5$ GeV and a distance $(731278.0 \pm 0.2)m$ from CERN-CNGS to the detector in Gran Sasso. This corresponds to a neutrino superluminality by the amount [1],

$$\delta(E = 12.5\text{GeV}) = \frac{v_\nu - c}{c} < 2.3 \times 10^{-7} \quad (3)$$

From (2) and (3) we put constraints on ξ_n from the tof experiment [2].

The Cerenkov decay constraint comes from the earlier ICARUS experiment [8] where the expected number of CC neutrino events is 315 ± 5 and the observed number is 308. This corresponds to a bound on the decay length $c\tau$ of neutrinos given by

$$0.04 < \exp(-c\tau/731.2\text{km}). \quad (4)$$

We calculate the neutrino Cerenkov decay length $c\tau$ using the generalised dispersion relations (1) and constraint ξ_n from the experimental bound (4).

The upper bounds on the Lorentz violating parameter ξ_n for different n obtained from the tof experiment [2] and Cerenkov decay experiment [8] are displayed in Table 1.

n	Upper bound on ξ_n	
	Time of flight [2]	Cerenkov process[8]
2	4.6×10^{-7}	1.7×10^{-5}
3	$5.9 \times 10^{-8} \text{ GeV}^{-1}$	$1.3 \times 10^{-6} \text{ GeV}^{-1}$
4	$3.8 \times 10^{-10} \text{ GeV}^{-2}$	$1.0 \times 10^{-7} \text{ GeV}^{-2}$
5	$2.5 \times 10^{-11} \text{ GeV}^{-3}$	$7.9 \times 10^{-9} \text{ GeV}^{-3}$
6	$1.7 \times 10^{-12} \text{ GeV}^{-4}$	$6.3 \times 10^{-10} \text{ GeV}^{-4}$
7	$1.2 \times 10^{-13} \text{ GeV}^{-5}$	$5.0 \times 10^{-11} \text{ GeV}^{-5}$

Table 1: Upper bounds on the Lorentz violating parameter ξ_n for different n obtained from the tof experiment [2] and neutrino Cerenkov decay experiment [8]

2 $\nu_\mu \rightarrow \nu_\mu e^+ e^-$

We compute the process $\nu_\mu(p) \rightarrow \nu_\mu(p')e^+(k)e^-(k')$ for GeV energy neutrinos. We will use the dispersion relations (1) for neutrinos in the Lab frame, and we will assume that all other particles have the standard energy-momentum relations, and we will assume energy momentum conservation in all reference frames. This generalises the Glashow-Cohen calculation for the same process for the $n = 2$ energy independent δ case.

The amplitude squared for the process is given by

$$\overline{|M|^2} = 32G_F^2 [(p \cdot k')(p' \cdot k)(1 - 4\sin^2 \theta_W + 8\sin^4 \theta_W)]. \quad (5)$$

The decay rate of the neutrino is in general given by

$$\Gamma = \frac{1}{8(2\pi)^5} \int \frac{d^3 p'}{E'_\nu} \frac{d^3 k'}{E'_e} \frac{\overline{|M|^2}}{E_\nu} \delta((p - p' - k')^2) \quad (6)$$

Without loss of generality we can choose

$$\begin{aligned} p &= (E_\nu, 0, 0, |\mathbf{p}|) \\ p' &= (E'_\nu, |\mathbf{p}'| \sin \theta, 0, |\mathbf{p}'| \cos \theta) \\ k' &= E'_e(1, \cos \phi \sin \theta_1, \sin \phi \sin \theta_1, \cos \theta_1). \end{aligned}$$

The argument of the δ function in eq.(6) can be as

$$(p - p' - k')^2 = \xi_n (p^{n-1} - (p')^{n-1}) (p - p') - pp'\theta^2 - E'_e D$$

where

$$D = \xi_n (p^{n-1} - (p')^{n-1}) - p' (\theta^2 + \theta_1^2) + p\theta_1^2 + 2p'\theta\theta_1 \cos\phi \quad (7)$$

Here we will assume that the transverse energy is very small and of the order of $\xi_n p^{n-2}$, therefore we keep only the leading order terms in θ , θ_1 and ξ_n . From here on we shall use the notation p and p' to denote $|\mathbf{p}|$ and $|\mathbf{p}'|$.

Now to fix the limits of the θ^2 and θ_1^2 integrals we need to find their maximum values from the δ -function condition i.e

$$DE'_e = \xi_n (p^{n-1} - (p')^{n-1}) (p - p') - pp'\theta^2 \quad (8)$$

For the maximum value of θ we set $E'_e = 0$ in the above equation so that have

$$\theta_{max}^2 = \frac{\xi_n (p^{n-1} - (p')^{n-1}) (p - p')}{pp'} \quad (9)$$

And similarly setting $p' = 0$ and the electron energy at its maximum i.e $E'_e = p/2$ in the δ -function condition we have,

$$(\theta_1^2)_{max} = \xi_n p^{n-2} \quad (10)$$

We make the following change of variables to pull out the factors of ξ_n and p from the integrand :

$$p' \rightarrow x p, \theta \rightarrow \xi_n p^{n-2} \tilde{\theta}, \theta_1 \rightarrow \xi_n p^{n-2} \tilde{\theta}_1$$

Using the above definitions in eq.(5) and substituting in eq.(6) gives the rate of electron-positron pair emission as

$$\Gamma = \frac{G_F^2}{16\pi^4} (\xi_n p^{n-2})^3 p^5 (1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W) \int_0^1 dx \int_0^{\frac{(1-x)(1-x^3)}{x}} d\tilde{\theta} \int_0^1 d\tilde{\theta}_1 \int_0^{2\pi} d\phi f(x, \tilde{\theta}, \tilde{\theta}_1, \phi) \quad (11)$$

where f is complicated function of x , $\tilde{\theta}$ and $\tilde{\theta}_1$ [42].

After numerically solving this integral we get the following expression for the rate of electron-positron pair emission the general formula for the decay width of neutrino splitting process $\nu(p) \rightarrow \nu(p')e^+(k)e^-(k')$ comes out to be

$$\Gamma = \frac{G_F^2}{16\pi^4} (1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W) I_n [\xi_n p^{n-2}]^3 p^5 \quad (12)$$

In eq.(12) I_n is an integral of a function depending on n . The values of I_n and Γ for different values of n are given in Table 2. Using $c\tau$ for different n

n	2	3	4	5	6	7	8
I_n	1/40	1/31	1/29	1/28	1/28	1/28	1/28

Table 2: Values of integral I_n in eqn.(12) for different values of n and for $E_\nu = 12.5\text{GeV}$.

and comparing with the experimental bound from ICARUS[8] shown in (4) we obtain the bounds on the Lorentz violating parameter ξ_n for different n displayed in Table 1.

This generalises our earlier calculation of the $n = 2$ and $n = 4$ cases [42]. Exact analytical calculations for the $n = 2$ case has been done [3, 42, 43, 44, 45]. Our result for the decay width is smaller than the corresponding result of [3] and [44] by a factor of 2/3 but is in closer agreement with the results of [43, 45].

3 Pion Decay

We calculate the pion decay width in the lab frame with a superluminal neutrino in the final state. We assume the dispersion relation $E^2 = (p^2 + \xi_n p^n)$ in the lab frame. The amplitude squared for the process $\pi^-(q) \rightarrow \mu^-(p)\bar{\nu}_\mu(k)$ is,

$$\overline{|M|^2} = 4G_F^2 f_\pi^2 m_\mu^2 \left[m_\pi^2 - m_\mu^2 + \xi_n k^n \left(\frac{m_\pi^2}{m_\mu^2} + 2 \right) \right] \quad (13)$$

The decay width is then given by

$$\Gamma = \frac{G_F^2 f_\pi^2 m_\mu^2}{8\pi E_\pi} \int \frac{k dk d\cos\theta}{\sqrt{|\vec{q} - \vec{k}|^2 + m_\mu^2}} \delta(E_\nu + \sqrt{|\vec{q} - \vec{k}|^2 + m_\mu^2} - E_\pi) \left[m_\pi^2 - m_\mu^2 + \xi_n k^n \left(\frac{m_\pi^2}{m_\mu^2} + 2 \right) \right] \quad (14)$$

Writing $|\vec{q} - \vec{k}|^2 = k^2 + q^2 - 2kq \cos\theta$, where θ is the angle between \vec{k} and \vec{q} , and $E_\nu = k + \xi_n k^{n-1}/2$ we see from the argument of the δ -function in eq.(14)

$$\cos\theta = (m_\mu^2 - m_\pi^2 + 2E_\pi k + \xi_n k^{n-1} E_\pi - \xi_n k^n) (2kq)^{-1} \quad (15)$$

while the derivative of the argument of δ -function with respect to $\cos\theta$ yields

$$\left| \frac{d}{d\cos\theta} (E_\nu + \sqrt{|\vec{q} - \vec{k}|^2 + m_\mu^2} - E_\pi) \right| = \frac{kq}{\sqrt{k^2 + q^2 - 2kq \cos\theta + m_\mu^2}} \quad (16)$$

p_π	Γ/Γ_0				
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
10 GeV	0.95	1.0	1.0	1.0	1.0
50 GeV	0.16	0.48	0.62	0.69	0.73
100 GeV	0.04	0.18	0.27	0.32	0.35
200 GeV	0.01	0.06	0.11	0.13	0.15
500 GeV	0.002	0.02	0.03	0.04	0.05

Table 3: Ratio of pion decay width in Lorentz violating framework to its SM prediction (Γ_0) at different values of pion momentum (p_π) for various n .

Substituting this in eq.(14) we get

$$\Gamma = \frac{G_F^2 f_\pi^2 m_\mu^2}{8\pi E_\pi} \int \frac{dk}{q} \left[m_\pi^2 - m_\mu^2 + \xi_n k^n \left(\frac{m_\pi^2}{m_\mu^2} + 2 \right) \right] \quad (17)$$

The limits of the k integral are fixed by taking $\cos \theta = \pm 1$ in eq.(15)

$$k_{max} = \frac{m_\pi^2 - m_\mu^2 - \xi_n k_{max}^{n-1} (E_\pi - k_{max})}{2(E_\pi - q)} \quad (18)$$

$$k_{min} = \frac{m_\pi^2 - m_\mu^2 - \xi_n k_{min}^{n-1} (E_\pi - k_{min})}{2(E_\pi + q)}$$

we solve these polynomial equations for k_{max} and k_{min} numerically to obtain the kinematically allowed limits of neutrino momentum. Using these limits to integrate over the neutrino momentum k we get the decay rate for pion. The ratio of pion decay rate thus calculated to the standard model prediction

$$\Gamma_0(\pi \rightarrow \mu\nu) = \frac{m_\pi^2}{E_\pi} \left(1 - \frac{m_\mu^2}{m_\pi^2} \right)^2 \quad (19)$$

for different n is shown in the Table 3. The $n = 4$ case has also been dealt with in [46] and we are in broad agreement with their result. In Fig. 1 an approximate numerical calculation of the pion decay width is plotted as a function of pion momentum for different n .

The decay width for the 100 GeV pions decreases by 65% for $n = 6$ and is smaller at higher n .

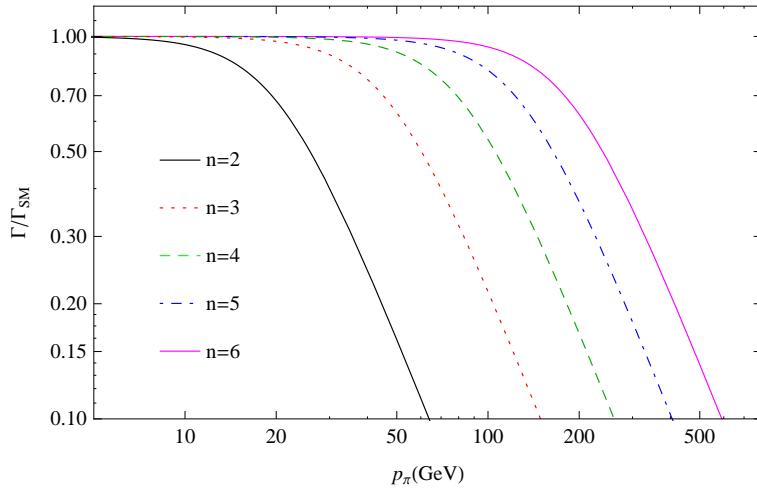


Figure 1: Ratio of pion decay width in Lorentz violating framework (approximate numerical calculation) to its Standard Model prediction as a function of pion momentum (p_π) for different n .

4 Conclusions

We have computed neutrino processes assuming a power law correction to the neutrino energy-momentum relations. We conclude that for steep power law ($n \geq 2$) dispersion relations the constraint from the time of flight experiments is more stringent than from the measurement of the Cerenkov e^+e^- emission or from the change in the width of pion decay. Our calculation of neutrino Cerenkov emission and pion decay width in Lorentz violating theories can be applied for putting bounds on Lorentz violating parameters from the analysis of high energy cosmic rays. Also, future experiments for measuring neutrino velocities performed at higher energies will put strong constraints on the higher derivative Lorentz violation theories [47].

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